TRINITY COLLEGE

IN NOMINE DOMINI

Semester Two Examination, 2023

Question/Answer booklet

MATHEMATICS METHODS UNITS 3&4

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

In words

Your name

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(8 marks)**

The manager of a high street bank wants to know what proportion of their customers never use cash and has asked an employee to collect sample data by standing in the bank foyer for two hours before lunch on a Thursday morning and questioning as many people as they can.

(a) Identify and explain two possible sources of bias with this sampling procedure. (4 marks)

Solution
The sample is at a fixed time – only customers who visit the bank on that day and at that time will be sampled, leading to undercoverage. The sample is at a fixed location – only customers who actually visit the bank will be sampled, again leading to undercoverage of all other customers.
Specific behaviours
✓ identifies a source of bias ✓ explains how the source introduces bias ✓ identifies a second source of bias ✓ explains how the second source introduces bias

(b) Briefly describe a sampling procedure that the manager could use in order to minimise all sources of bias. (2 marks)

Solution
Randomly select individuals from a list of all bank customers and contact them by their preferred means to obtain their response to the question.
Specific behaviours
✓ indicates use of random sampling ✓ indicates random sample to be drawn from all customers

From the 225 responses obtained using a reliable sampling procedure, the manager was presented with the confidence interval $(0.1924, 0.2876)$ for the proportion of their customers who never use cash.

(c) Determine the number of customers in the sample who said they never use cash. (2 marks)

Solution
$p = (0.1924 + 0.2876) \div 2 = 0.24$ $n = 0.24 \times 225 = 54$ customers.
Specific behaviours
✓ indicates correct sample proportion ✓ correct number of customers

Question 9 **(7 marks)**

The air pressure, P kPa, inside the tyre of a motor vehicle t seconds after it was punctured can be modelled by the equation $P = a + 122e^{kt}$, where a and k are constants.

The initial pressure in the tyre was 220 kPa and after 8.5 seconds it had dropped to 142 kPa.

(a) Determine the value of a and the value of k. (3 marks)

Solution
$P(0) = a + 122 = 220 \Rightarrow a = 98$ $P(2) = 98 + 12e^{8.5k} = 142 \Rightarrow k = -0.12$
Specific behaviours
✓ correct value of a ✓ forms equation for k ✓ correct value of k

(b) Determine

(i) the pressure in the tyre after 5 seconds. (1 mark)

Solution
$P = 98 + 122e^{-0.12\times 5} = 165.0\text{ kPa}$
Specific behaviours
✓ correct pressure

(ii) the time taken for the pressure in the tyre to fall to 99 kPa. (1 mark)

Solution
$99 = 98 + 122e^{-0.12t} \Rightarrow t = 40.0\text{ s}$
Specific behaviours
✓ correct time

(c) Given that the pressure was falling at a rate of 8 kPa per second after 5 seconds, use the increments formula to estimate the pressure in the tyre after 5.1 seconds. (2 marks)

Solution
$\delta P \approx \dfrac{dP}{dt}\delta t$ $\approx -8 \times 0.1 = -0.8$ $P = 165 - 0.8 = 164.2\text{ kPa}$
Specific behaviours
✓ uses increments formula to estimate change   ✓ adds change to previous pressure to obtain estimate

Question 10 **(8 marks)**

Liu has a hen that on some days lays an egg and on other days it doesn't. The random variable X is the number of eggs that this hen lays in a day, so that

$$P(X = x) = \begin{cases} a(3x + 2) & x = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of the constant a and hence state the probability that Liu's hen does not lay an egg in a day. (2 marks)

Solution
$P(X = 0) = 2a, \quad P(X = 1) = 5a, \quad 7a = 1, \quad a = \frac{1}{7}$ Hence $P(X = 0) = \frac{2}{7}$.
Specific behaviours
✓ indicates $P(X = 0)$ and $P(X = 1)$ in terms of a ✓ correct probability

Liu also sells jars of honey from a roadside honesty box in which she places 4 jars every morning. The random variable Y is the number of jars of honey Liu sells in a day so that

$$P(Y = y) = \begin{cases} k(y - 1.5)^2 & y = 0, 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

(b) Determine the exact value of the constant k. (2 marks)

Solution
$\Sigma\big(P(Y = y)\big) = k(2.25 + 0.25 + 0.25 + 2.25 + 6.25) = 11.25k$ Hence $k = 1 \div 11.25 = \frac{4}{45}$.
Specific behaviours
✓ indicates correct method ✓ states exact value of k

Assume that X and Y are independent and that Liu has a good day when her hen lays an egg and she sells at least 2 jars of honey.

(c) Determine the probability that Liu has a good day. (2 marks)

Solution
$P(X = 1) \times P(Y \geq 2) = \frac{5}{7} \times \frac{4}{45}(11.25 - 2.25 - 0.25) = \frac{5}{7} \times \frac{7}{9} = \frac{5}{9} = 0.\overline{5}$
Specific behaviours
✓ indicates correct probability for $P(Y \geq 2)$ ✓ correct probability

(d) Determine the probability that Liu has exactly 6 good days in a week. (2 marks)

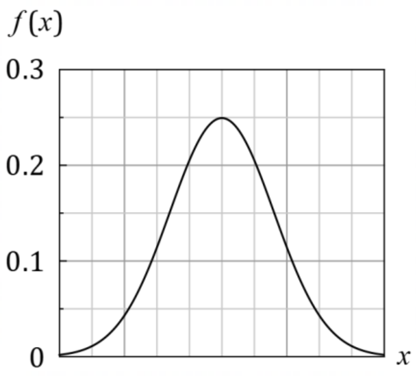
Solution
$W \sim B\left(7, \frac{5}{9}\right), \quad P(W = 6) = 0.0915$
Specific behaviours
✓ indicates correct binomial distribution ✓ correct probability

Question 11 **(6 marks)**

(a) The probability density function f of a normal distribution with mean μ and standard deviation σ is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The graph of a normal distribution with a mean of 18 is shown below. Estimate, to one decimal place, the standard deviation of the distribution. Justify your answer. (2 marks)



Solution
$f(\mu) = \frac{1}{\sigma\sqrt{2\pi}}\, e^0 \Rightarrow \frac{1}{\sigma\sqrt{2\pi}} = 0.25$ Hence $\sigma = 1 \div \sqrt{2\pi} \div 0.25 = 1.596 = 1.6$ (1 dp).
Specific behaviours
✓ uses $f(\mu) = 0.25$ to obtain equation for σ ✓ correct estimate, with justification

(b) The weights, W grams, of eggs produced by the free-range hens at a small farm are normally distributed with a mean of 58.6 g and a standard deviation of 5.8 g.

Determine the probability that a randomly selected egg from this farm

(i) weighs more than 65 g. (1 mark)

Solution
$P(W > 65) = 0.1349$
Specific behaviours
✓ correct probability

(ii) weighs no more than 55 g. (1 mark)

Solution
$P(W < 55) = 0.2674$
Specific behaviours
✓ correct probability

(iii) weighs less than 55 g, given that it weighs less than 65 g. (2 marks)

Solution
$P(W < 55 \mid W < 65) = \frac{P(W < 55)}{P(W < 65)}$ $= \frac{0.2674}{1 - 0.1349} = \frac{0.2674}{0.8651} = 0.3091$
Specific behaviours
✓ shows correct denominator   ✓ correct probability

Question 12 **(10 marks)**

In the Shire of Murchison, 65% of the feral goat population is female.

(a) Use a discrete probability distribution to determine the probability that when a random sample of 50 feral goats is taken, no more than 60% of them will be female. (3 marks)

Solution
$X \sim \mathrm{B}(50, 0.65)$, $x \leq 0.6 \times 50 = 30$, $\mathrm{P}(X \leq 30) = 0.2736$
Specific behaviours
✓ indicates use of binomial distribution with correct parameters ✓ indicates number of successes required ✓ correct probability

A large batch of simulations are run in which a random sample of 50 feral goats is selected from the Murchison population and the proportion of females in the sample calculated.

(b) Describe the continuous probability distribution that the sample proportions from the simulations will approximate. (2 marks)

Solution
Distribution will be normal with mean 0.65 and variance $\dfrac{0.65 \times 0.35}{50} = 0.00445$. (NB $\mathrm{sd} \approx 0.0675$)
Specific behaviours
✓ states normal distribution with correct mean ✓ states correct variance or standard deviation

Another simulation is run to obtain one more sample proportion.

(c) Use the distribution from part (b) to determine:

(i) the probability that the sample proportion will be no more than 60%. (1 mark)

Solution
$\mathrm{P}(X \leq 0.6) = 0.2293$
Specific behaviours
✓ correct probability

(ii) the value of the constant k, given that there is an 90% chance that the sample proportion lies between 0.55 and k. (2 marks)

Solution
$P(0.55 < X < k) = 0.9$, $k = 0.7760$
Specific behaviours
✓ correct probability statement or diagram ✓ correct value of k

It is thought that for older (over the age of 11) feral goats from this region, that 88% are female.

A second batch of simulations are run in which a random sample of 25 older feral goats is selected from the Murchison population and the proportion of females in the sample calculated.

(d) State two reasons that the distribution of the sample proportions from this second batch will not yield such a good approximation to the type of distribution described in part (b) when compared to the first batch. (2 marks)

Solution
The sample size in the second batch is smaller than the first, and a larger sample size yields a better approximation. The proportion in the second batch is closer to 1 than the first batch and so the second sampling distribution will display much more left skew. The value of $n(1-p) = 25 \times 0.12 = 3$, but this should be at least 5* for a good approximation. ** Syllabus dot point 4.3.6 states that the closeness of the approximation depends on both n and p but the glossary does not expand on this.* 15 *seems to be the currently preferred minimum for np and $n(1-p)$, but accept the traditional value of 5 (or more).*
Specific behaviours
✓ states a valid reason involving n ✓ states a valid reason involving p

Question 13 **(10 marks)**

Members of a toy library may take home up to 5 toys per visit. The following frequency table shows the number of toys borrowed by a random sample of 80 members.

Toys borrowed	0	1	2	3	4	5
Frequency	0	10	26	28	12	4

You may assume that relative frequencies obtained from the above data are reliable point estimates of probabilities and that the number of toys borrowed by any two members are independent.

(a) Determine the probability that a member borrows fewer than 5 toys, given that they borrowed at least 3 toys. (2 marks)

Solution
$p = \dfrac{28+12}{28+12+4} = \dfrac{40}{44} = \dfrac{10}{11} \approx 0.9091$
Specific behaviours
✓ correct denominator ✓ correct probability

(b) Determine the probability that at least 3 of the next 5 borrowers take home an even number of toys. (3 marks)

Solution
$Y \sim \mathrm{B}(5, p),\quad p = \dfrac{26+12}{80} = \dfrac{38}{80} = 0.475$ $P(Y \geq 3) = 0.4532$
Specific behaviours
✓ indicates use $\mathrm{B}(5, p)$ ✓ indicates correct value of p ✓ correct probability

(c) Show that the mean of the random variable X, the number of toys borrowed by a member, is 2.675 and determine the variance of X. (3 marks)

Solution
$\mathrm{E}(X) = 1 \times 0.125 + 2 \times 0.325 + 3 \times 0.35 + 4 \times 0.15 + 5 \times 0.05$ $= 0.125 + 0.65 + 1.05 + 0.6 + 0.25$ $= 2.675$ $\mathrm{Var}(X) = 1.069375 \approx 1.0694$
Specific behaviours
✓ shows correct expression for mean ✓ shows result of each product and their sum ✓ correct variance

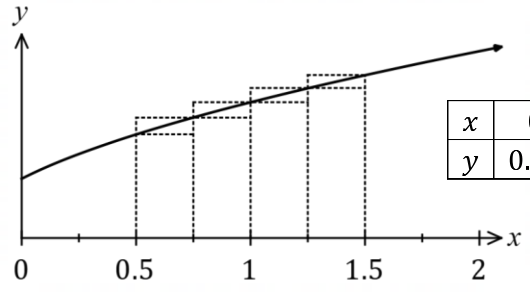
Observations indicate that members spend 7 minutes at the library plus 3 minutes per toy chosen.

(d) Determine the mean and standard deviation of the random variable T, the time in minutes spent by members at the toy library. (2 marks)

Solution
$T = 3X + 7$ $\mathrm{E}(T) = 3(2.675) + 7 = 15.025$ minutes $\mathrm{sd}_T = 3 \times \sqrt{1.069375} \approx 3.102$ minutes
Specific behaviours
✓ correct mean ✓ correct standard deviation

Question 14 **(8 marks)**

The graph of $y = f(x)$ and a table of values for the function f are shown below.



x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
y	0.96	1.38	1.68	1.94	2.16	2.42	2.64	2.84	3.03

(a) By considering the areas of the rectangles shown, demonstrate and explain why 2.17 is a reasonable estimate for $\int_{0.5}^{1.5} f(x)\,dx$. (3 marks)

Solution
Area of inscribed rectangles: $A_I = 0.25(1.68 + 1.94 + 2.16 + 2.42) = 2.05$ Area of circumscribed rectangles: $A_C = 0.25(1.94 + 2.16 + 2.42 + 2.64) = 2.29$ Average of underestimate and overestimate: $(2.05 + 2.29) \div 2 = 2.17$
Specific behaviours
✓ shows calculation for inscribed rectangles   ✓ shows calculation for circumscribed rectangles   ✓ indicates one is underestimate, other is overestimate and averages

(b) Determine, with justification, estimates for

(i) $\int_{0.5}^{1.5} 4f(x)\,dx$. (1 mark)

Solution
$I = 4\int_{0.5}^{1.5} f(x)\,dx = 4 \times 2.17 = 8.68$
Specific behaviours
✓ correct estimate, shows product

(ii) $\int_{0.5}^{1.5} f(x) + 4\,dx$. (2 marks)

Solution
$I = \int_{0.5}^{1.5} f(x)\,dx + \int_{0.5}^{1.5} 4\,dx = 2.17 + 4 = 6.17$
Specific behaviours
✓ indicates use of linearity to split integral   ✓ correct estimate

(iii) $\int_{0}^{1} f\left(\frac{x}{2}\right) dx$. (2 marks)

Solution
$A_I = 0.5(0.96 + 1.38) = 1.17$ $A_C = 0.5(1.38 + 1.68) = 1.53$ $I = (1.17 + 1.53) \div 2 = 1.35$
Specific behaviours
✓ indicates use of appropriate intervals   ✓ correct estimate

Question 15 **(8 marks)**

Repair tasks undertaken by technical staff who work at an IT company are assigned minor, major or critical status. Over the long term, 4% of the tasks have been critical, 24% major and the remainder minor.

(a) Assuming that the long-term proportions are correct, determine the smallest sample size required so that the width of a 90% confidence interval for the proportion of minor tasks is less than 0.093. (3 marks)

Solution
$p = 1 - 0.04 - 0.24 = 0.72, \qquad E = 0.093 \div 2 = 0.0465$ $$n > \frac{1.645(0.72)(0.28)}{0.0465^2} = 252.3$$ A sample size of 253 is required.
Specific behaviours
✓ indicates correct margin of error   ✓ uses appropriate formula   ✓ correct smallest sample size

At the end of one month, a manager suspects that the proportion of major tasks has changed and so she takes a random sample of 250 tasks from the last month, of which 45 were major.

(b) Use this sample to construct a 95% confidence interval for the proportion of major tasks. (3 marks)

Solution
$\hat{p} = 45 \div 250 = 0.18$ $$0.18 \pm 1.96\sqrt{\frac{0.18(0.82)}{250}} = 0.18 \pm 0.0476$$ The 95% confidence interval is $(0.1324, 0.2276)$.
Specific behaviours
✓ indicates correct sample proportion   ✓ indicates correct margin of error   ✓ correct interval

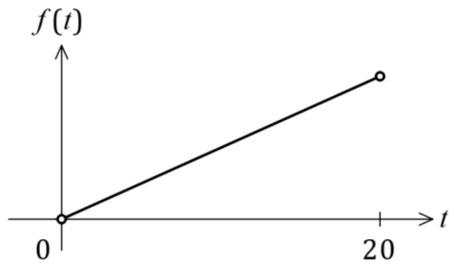
(c) Does your confidence interval in part (b) support the managers suspicions? Justify your answer. (2 marks)

Solution
Yes – the interval does not contain the long-term proportion of 24% and so the sample supports the hypothesis that the proportion of major tasks has changed.
Specific behaviours
✓ indicates interval supports claim   ✓ states interval does not contain long-term value

Question 16 **(8 marks)**

Alan works from home every Tuesday and always starts work after his digital clock first shows 8: 10 am and before it shows 8: 30 am.

The probability density function for T, the time in minutes after 8: 10 that he starts work, is $f(t)$ and is displayed at right.



(a) Write the defining rule for the probability density function $f(t)$. (2 marks)

Solution

Let $f(t) = kt$ then:

$$\int_0^{20} kt\,dt = 1 \Rightarrow 200k = 1 \Rightarrow k = \frac{1}{200} = 0.005$$

$$f(t) = \begin{cases} t/200 & 0 < t < 20 \\ 0 & \text{Otherwise} \end{cases}$$

Specific behaviours

✓ indicates area beneath $f(t)$ must be 1
✓ correct function (piecewise form / otherwise not required)

(b) Determine the probability that on a randomly chosen Tuesday, Alan starts work after his clock first shows 8: 23 am. (2 marks)

Solution

$$\int_{13}^{20} \frac{t}{200}\,dt = \frac{231}{400} = 0.5775$$

Specific behaviours

✓ writes correct integral with bounds
✓ correct probability

(c) Determine the mean and standard deviation of T. (4 marks)

Solution

$$\bar{T} = \int_0^{20} \frac{t^2}{200}\,dt = \frac{40}{3} = 13.\bar{3}$$

$$\text{Var}(T) = \int_0^{20} \left(\left(t - \frac{40}{3}\right)^2 \times \frac{t}{200}\right) dt = \frac{200}{9} = 22.\bar{2}, \qquad \sqrt{\frac{200}{9}} = \frac{10\sqrt{2}}{3} \approx 4.714$$

Mean of T is $13.\bar{3}$ minutes and standard deviation is 4.714 minutes.

Specific behaviours

✓ correct integral for mean
✓ correctly evaluates mean
✓ correct integral for variance
✓ correctly evaluates standard deviation

Question 17 **(8 marks)**

The acceleration a m/s^2 of a train moving in a straight line at time t seconds is given by

$$a = k + \frac{1}{4}\cos\left(\frac{\pi t}{10}\right), \qquad 0 \le t \le 60.$$

Initially, the train was at an origin O and moving with a velocity of 4 m/s.

(a) Determine the velocity of the train after 10 seconds when the constant $k = 1.5$. (3 marks)

Solution
$v(t) = \int a(t)\,dt = 1.5t + \frac{5}{2\pi}\sin\left(\frac{\pi t}{10}\right) + c$ $v(0) = 4 \Rightarrow c = 4$ $v(10) = 1.5(10) + \frac{5}{2\pi}\sin(\pi) + 4 = 19\ \mathrm{m/s}$
Specific behaviours
✓ determines antiderivative of a ✓ evaluates constant of integration   ✓ correct velocity

After 20 seconds the displacement of the train relative to the origin O was 200 m.

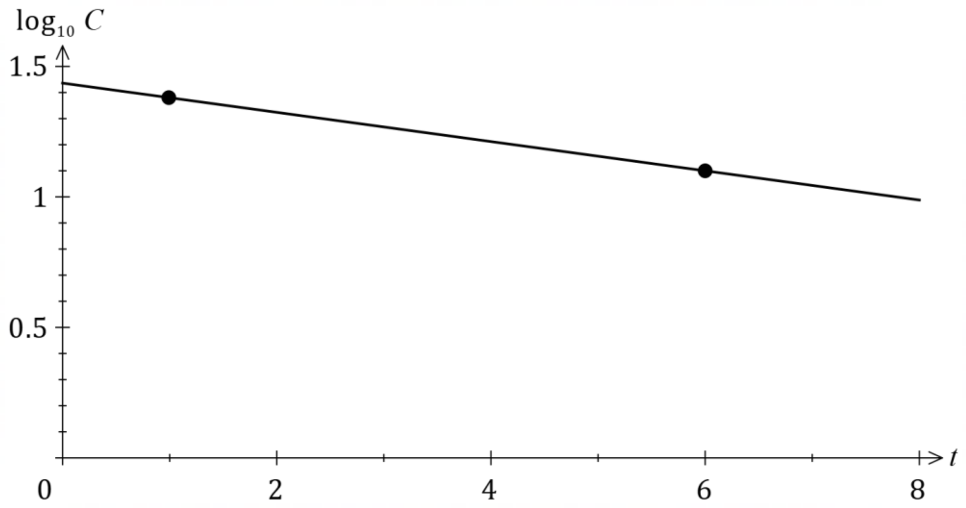
(b) Determine the value of the constant k and hence calculate, to the nearest metre, the displacement of the train after 35 seconds. (5 marks)

Solution
$v = \int a(t)\,dt = kt + \frac{5}{2\pi}\sin\left(\frac{\pi t}{10}\right) + c, \qquad v(0) = 4 \Rightarrow c = 4$ $v(t) = kt + \frac{5}{2\pi}\sin\left(\frac{\pi t}{10}\right) + 4$ $s(t) = \frac{kt^2}{2} - \frac{25}{\pi^2}\cos\left(\frac{\pi t}{10}\right) + 4t + C, \qquad s(0) = 0 \Rightarrow C = \frac{25}{\pi^2}$ $s(20) = 200 \Rightarrow \frac{k(20)^2}{2} - \frac{25}{\pi^2}\cos(2\pi) + 4(20) + \frac{25}{\pi^2} = 200 \Rightarrow k = \frac{3}{5} = 0.6$ $s(35) = \frac{3}{5} \times \frac{(35)^2}{2} - \frac{25}{\pi^2}\cos\left(\frac{35\pi}{10}\right) + 4(35) + \frac{25}{\pi^2} = \frac{25}{\pi^2} + \frac{1015}{2} \approx 510\ \mathrm{m}$
Specific behaviours
✓ uses result from (a) to obtain $v(t)$ ✓ determines antiderivative of $v(t)$ ✓ evaluates constant of integration and forms equation for k ✓ evaluates k ✓ correct displacement

Question 18 **(10 marks)**

The concentration C $\mu\text{g/mL}$ of a pain-killing drug in a patient was observed for several hours.

The graph of $\log_{10} C$ against time t hours is linear and is shown below, passing through the points $(1, 1.38)$ and $(6, 1.1)$.



The relationship between C and t can be written in the form $\log_{10} C = at + b$.

(a) Determine the value of the constant a and the value of the constant b. (2 marks)

Solution
$a = \frac{1.1 - 1.38}{6 - 1} = -\frac{0.28}{5} = -0.056$ $b = 1.38 - (-0.056) \times 1 = 1.436$
Specific behaviours
✓ correct gradient   ✓ correct intercept

(b) Determine where the straight line intercepts the horizontal axis and interpret what this point represents in the context of this question. (3 marks)

Solution
$-0.056t + 1.436 = 0 \Rightarrow t = 25.64$ $\log_{10} C = 0 \Rightarrow C = 1$ The horizontal intercept is at $(25.64, 0)$.   The intercept can be interpreted to mean that after 25.64 hours, the drug concentration has fallen to 1 $\mu\text{g/mL}$.
Specific behaviours
✓ correct value of t for horizontal intercept   ✓ indicates value of C at this time   ✓ correctly interprets in context

(c) Show that C can be expressed as a function of t in the form $C = k(10)^{at}$ and state the value of the constant k. (3 marks)

Solution
$\log_{10} C = -0.056t + 1.436$ $C = 10^{-0.056t+1.436}$ $= 10^{1.436} \times 10^{-0.056t}$ $= 27.3(10)^{-0.056t}$ Hence $k = 27.3$.
Specific behaviours
✓ correctly eliminates the log term ✓ uses index laws to separate 10^{at+b} ✓ states correct value of k

(d) Determine

(i) the concentration of the drug in the patient after 9 hours. (1 mark)

Solution
$C = 27.3(10)^{-0.056(9)}$ $= 8.55\ \mu\text{g/mL}$
Specific behaviours
✓ correct concentration

(ii) the time taken for the concentration of the drug to first fall below 5% of its initial value. (1 mark)

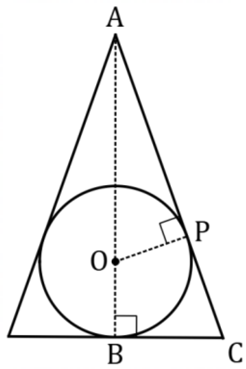
Solution
$10^{-0.056t} = 0.05$ $t = 23.23\ \text{h}$
Specific behaviours
✓ correct time

Question 19 **(7 marks)**

A right-cone of radius $BC = r$ cm, height $AB = h$ cm and volume V cm^3 is circumscribed about a sphere of radius $OB = OP = 9$ cm.

The diagram shows a cross-section through the centre of the cone and sphere.

Note that $\Delta ABC \sim \Delta APO$.



(a) Show that $V = \dfrac{27\pi h^2}{h-18}$. (3 marks)

Solution
Using similar triangles: $$\frac{AB}{BC} = \frac{AP}{PO} \Rightarrow \frac{h}{r} = \frac{\sqrt{(h-9)^2 - 9^2}}{9}$$ $$r = \frac{9h}{\sqrt{h^2 - 18h}}$$ Volume of cone: $$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{81h^2}{h^2 - 18h}\right)h = \frac{27\pi h^2}{h-18}$$
Specific behaviours
✓ correctly uses similar triangles to relate r and h ✓ obtains simplified expression for r in terms of h ✓ substitutes for r^2 in volume formula and simplifies

(b) Use calculus to determine the minimum possible volume of the right-cone. (4 marks)

Solution
$$V'(h) = \frac{27\pi h^2 - 972\pi h}{(h-18)^2}$$ $$V'(h) = 0 \Rightarrow h = 0, 36$$ Note that if $h = 0$ then r is undefined and so discard this possibility. $$V(36) = 1944\pi$$ Since $V''(36) = 3\pi > 0$ then minimum when $h = 36$. The minimum volume is $1944\pi \approx 6107\ \text{cm}^3$.
Specific behaviours
✓ correct derivative ✓ correctly determines roots of derivative ✓ justifies $h = 36$ is a minimum ✓ correct minimum volume

End of questions

Supplementary page

Question number: _________